Gaussian local unitary equivalence of *n*-mode Gaussian states & Gaussian LOCC transformations

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Motivation and Outline

- multipartite entanglement is a mess
- ... too many classes, too few applications for which it is a resource
- generating multipartite entanglement (of individually accessible modes) is fairly straightforward in the conmtinuous variable setting [Silberhorn, Pfister, Furusawa, Schnabel, Treps, Peng, ...]
- single squeezed state and a beam-splitter array is enough; many squeezing processes are inherently multi-mode
- up to 10^4 mode entanglement demonstrated [Yokoyama 2013]
- Gaussian states are in many respect a very simple family, thanks to the direct-sum structure of phase space

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Entanglement Classes

- states ϕ, ψ represent essentially same resource if they can be reversibly interconverted by "available" (local) operations:

$\Rightarrow$ $\psi \overset{\text{LU}}{\sim} \phi$ iff $\psi = (\otimes_i U_i)\phi$ for local unitaries U_i
$\psi \overset{\text{SLOCC}}{\sim} \phi$ iff $\psi = (\otimes_i A_i)\phi$ for invertible A_i

- what can be learned?
- single-copy, $\dim\mathcal{H} < \infty$: *Schmidt coefficients* define LU classes, *Schmidt rank* defines SLOCC-classes;
- pure discrete multipartite states: 2 qubits: 2 SLOCC classes; 3 qubits: 6 SLOCC classes; 4 qubits: infinitely many...
  quite intricate: e.g. *n* qubits/LU: [Kraus PRL 2010]; *n* qudits/SLOCC: [Gour & Wallach PRL 2013]

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Entanglement of Multi-Mode Bosonic Gaussian States: some notation...

- symmetric, positive $2N \times 2N$ covariance matrix γ

$$\gamma \geq i\sigma, \text{ where } \sigma = i \oplus_{k=1}^{N} \sigma_y$$

- product state: $\gamma = \gamma_1 \oplus \gamma_2 \oplus \dots \gamma_n$
- Gaussian local unitary (GLU): $S = S_1 \oplus S_2 \oplus \dots S_n$, where $S_i \sigma S_k^T = \sigma \forall k$ (symplectic $Sp(2n_k)$)
- pure state: $\gamma\sigma\gamma = \sigma$ ($\gamma \in Sp(2N)$)
- Euler decomposition $S = O_1 Q O_2$, where $O_i \in SO(2N) \cap Sp(2N)$; $Q = \oplus_k \mathrm{diag}(q_k, 1/q_k), q_k > 0$
- Williamson form: $\gamma = SDS^T$, where $S \in Sp(2N), D = \oplus_k d_k \mathbb{1}_{2n_k}$

n-partite Gaussian States

- simplest case: one mode per party: $1 \times 1 \times \cdots \times 1$ states:

$$\gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{12}^T & \gamma_{22} & \cdots & \vdots \\ \vdots & & & \\ \gamma_{1n}^T & \cdots & \gamma_{n-1n} & \gamma_{nn} \end{pmatrix}$$

$\Rightarrow$ use GLU $\oplus S_k$ to bring γ to standard form $\mathcal{S}(\gamma)$ (defining its GLU-equivalence class)

1. pick S_i to symplectically diagonalize $\gamma_{jj} = \lambda_j \mathbb{1}$
2. *passive* local unitaries $O = O_1 \oplus \cdots \oplus O_n$ still undetermined ($O_j = e^{i\alpha_j \sigma_y} \in SO(2)$)
3. chose O_j, O_k to diagonalize $\gamma_{jk}, j < k$ (or $\gamma_{jk}^T \gamma_{jk}$) until all are fixed
4. generically: $\gamma_{12} = \mathrm{diag}(d_{12}, d'_{12}), \gamma_{1l}^T \gamma_{1l} = \mathrm{diag}(d_{1l}, d_{1l}), d_{kl} \geq |d'_{kl}|$
5. "degenerate cases $\gamma_{jk} = 0, \propto O, \propto \sigma_z O$ allow to simplify a more γ_{jk}

$\Rightarrow$ $n(2n-2)$ free parameters (for $n > 1$)

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GLU Standard Form and GLU Equivalence

Any $1 \times 1 \cdots \times 1$ CM γ can be brought to standard form $\mathcal{S}(\gamma)$ by GLU. Two CMs are GLU-equivalent iff they have the same standard form.

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- for pure states exploiting the constraint $\sigma\gamma\sigma = \gamma^{-1}$ allows further simplification (for small n [Adesso et al.])
- see also: Adesso et al, PRA 2006, 2007 (for generic states) some extensions to $n_1 \times n_2 \dots n_N$ [Adesso, Illuminati, Serafini, Wang, ...]

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Case Study: Pure Three-Mode States

- characterized by three real parameters $\lambda_1, \lambda_2, \lambda_3$ [Adesso PRA 2006]

$$\gamma(\lambda_1, \lambda_2, \lambda_3) = \begin{pmatrix} \lambda_1 \mathbb{1} & D_{12} & D_{13} \\ D_{12} & \lambda_2 & D_{23} \\ D_{13} & D_{23} & \lambda_3 \mathbb{1} \end{pmatrix}$$

$$\lambda_i + 1 \leq \lambda_j + \lambda_k \forall (ijk)$$

$D_{ij} = \mathrm{diag}(d_{ij}^+, d_{ij}^-)$ (function of the λ's)

- λ_i: local mixedness: measures entanglement of mode i with the other two

$\Rightarrow$ $\gamma(\lambda)$ more entangled than $\gamma(\lambda')$ if $\lambda_i \geq \lambda_i' \forall i$

★ states with different λ even belong to different **LU** classes (not just GLU!)

GLU vs. LU equivalence

- of course $\psi \overset{GLU}{\sim} \phi \implies \psi \overset{LU}{\sim} \phi$ (and the reverse in general false)

$\Rightarrow$ there could be GLU-nonequivalent Gaussian states that are, nevertheless, LU-equivalent

- I know of no example; and for pure $n \times m$ and $1 \times 1 \times 1$ this does not occur:
- GLU-classes are in 1:1 correspondence with λ_k, i.e., the Schmidt-coefficients (across different bipartitions)

  these are LU invariant, hence $\psi \overset{LU}{\sim} \phi$ implies $\psi \overset{GLU}{\sim} \phi$ for pure $n \times m, 1 \times 1 \times 1$ Gaussian states

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Some examples

- symmetric states $\lambda_j = \lambda \geq 1$,

$$d^{\pm} = \frac{1}{4\lambda}\left((\lambda^2 - 1) \pm \sqrt{9\lambda^4 - 10\lambda^2 + 1}\right)$$

- det $D_{ij} < 0 \forall (ij)$

? have been referred to as "GHZ-/W-state analogon" and as "maximally entangled" (they maximize certain tripartite ent measure *and* ent of two-mode reductions) [Adesso et al 2006]

- other degenerate cases: $D_{ij} \propto \mathbb{1}$ implies $\lambda_k = \lambda_i + \lambda_j - 1$, and thus $D_{ik}, D_{jk} \propto \sigma_z$ (and $D_{ij} \propto \sigma_z \implies$ one of the other $D \propto \mathbb{1}$)
  these states are obtained by coupling a two-mode squeezed state with the vacuum at a beam splitter $(\mathbb{1} \oplus B(\theta))(\gamma(r) \oplus \mathbb{1})(\mathbb{1} \oplus B(\theta))^T$
- in general: need TMSS $\gamma(r)$ and *three* beam splitters B_{13}, B_{23}, B_{13} [Adesso et al. 2007]

Maximal Entanglement?

? are the symmetric states $\lambda_i = \lambda$ in that we can locally generate *any* $(\lambda_1, \lambda_2, \lambda_3)$ from a suitable symmetric state?

$\Rightarrow$ need to go beyond GLU!

- include Gaussian measurements: adjoin a Gaussian ancilla, apply GLU, and perform Gaussian measurement on ancilla: deterministic Gaussian local operation + classical communication ("GLOCC")
- Gaussian measurement: POVM of projectors on Gaussian states $\{|\gamma, d\rangle\langle\gamma, d| : d \in \mathbb{C}^d\}$; e.g.: $\gamma = \mathbb{1}$: heterodyne detection (optimal joint measurement of X and P)

GLOCC Transformations

- Gaussian measurement transforms CM γ as [PRA 2002]

$$\gamma \mapsto \Gamma_1 - \Gamma_{12} \frac{1}{\Gamma_2 + \gamma} \Gamma_{12}^T$$

where $\Gamma = \begin{pmatrix} \Gamma_1 & \Gamma_{12} \\ \Gamma_{12}^T & \Gamma_2 \end{pmatrix}$ is pure CM (Choi-Jamiolkowski state)
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Reminder: Bipartite GLOCC-transformability

- all states GLU equivalent to product of two-mode squeezed states

$$\left|\gamma(\vec{r})\right\rangle \equiv \otimes_k \left|\gamma(r_k)\right\rangle \propto \otimes_k \sum_n \tanh^n(r_k) \left|n\right\rangle_A \otimes \left|n\right\rangle_B$$

- GLOCC allows only transformations to "less two-mode squeezing":

$$\gamma(\vec{r}) \stackrel{\text{GLOCC}}{\longrightarrow} \gamma(\vec{s}) \ \text{ iff } \ s_k \leq r_k \forall k$$

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GLOCC Trafos for Pure Three-mode States

? is transformation $(\lambda_1, \lambda_2, \lambda_3) \to (\lambda'_1, \lambda'_2, \lambda'_3)$ possible by GLOCC?

- sufficient to study how diagonal blocks change!
- we have only necessary conditions:
  - $\lambda'_i \leq \lambda_i \, \forall i$ (obviously)
  - $|D_{ij}| \leq 0 \forall (ij)$ implies $|D'_{ij}| \leq 0 \forall (ij)$
- what consequences? are there incomparable states?
  three possibilities: (i) $\gamma \to \gamma'$; (ii) $\gamma' \to \gamma$, or (iii) $\gamma \not\leftrightarrow \gamma'$

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- consider simple families of states $\gamma(\lambda_1, \lambda_2, \lambda_3)$ such as
  - *symmetric states* $\gamma_{\mathrm{symm}}(\lambda) = \gamma(\lambda, \lambda, \lambda)$
  - *shared two-mode squeezed states* $\gamma_{\mathrm{s-tmss}}(r, \theta)$

    (obtained by sending part of a two-mode squeezed state $\gamma(r)$ through a beam splitter with transmittivity $\cos^2\theta$)
- $\gamma_{\mathrm{symm}}(\lambda)$ has $|D_{ij}| \leq 0 \forall (ij)$ while $\gamma_{\mathrm{s-tmss}}(r, \theta)$ has $|D_{23}| > 0$

$\Rightarrow$ $\gamma_{\mathrm{symm}}(\lambda) \not\to \gamma_{\mathrm{s-tmss}}(r, \theta)$!

- converse *is* possible (for sufficiently large r)
  $\Longrightarrow$ $\{\gamma_{\mathrm{s-tmss}}(r, \theta) : r \geq 0\}$ is a "more entangled set" than $\{\gamma_{\mathrm{symm}}(\lambda) : \lambda \geq 0\}$ [cf. de Vincente et al., PRL 2013]
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Conclusions and Outlook

- provided standard form $\mathcal{S}(\gamma)$ for n-mode n-partite Gaussian states that allows to determine the GLU class of all such states
- showed that for pure three-mode states this leads to a nice parametrization by local mixednesses alone (as already shown by Adesso)
- seen that there is a nontrivial hierarchy between pure $1 \times 1 \times 1$ states in terms of their GLOCC-transformability

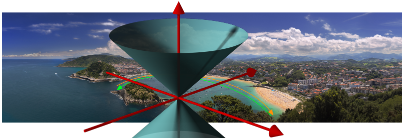
? is there a simple "maximal entangled family" (from which all others can be locally produced)?

? what is the meaning (if any) of $\det D_{12} > 0$ vs all three < 0? does it generalize to more modes per site?
does $\det D_{ij} \leq 0 \not\to \det D_{12} > 0$ hold for several copies?

? can we get coarser/more instructive classes by looking at multi-copy trafos? approximate trafos? LOCC conversion?

Thank you!

Giedke and Kraus, Phys. Rev. A **89** 012335 (2014)



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UPV EHU | CAMPUS OF INTERNATIONAL EXCELLENCE

TOPOLOGICAL MATTER SCHOOL 2016

August 22 - 26, 2016

List of Lecturers:

Mois Aroyo (UPV/EHU Bilbao)
Alexander Altland (University of Cologne)
Andrei Bernevig (Princeton University)
Claudia Felser (Max Planck Dresden)
Ingrid Mertig (University of Halle-Wittenberg)
Ivo Souza (CFM Donostia-San Sebastián)
Alexey Soluyanov (ETH Zürich)

Palacio Miramar, Donostia-San Sebastián

This one week summer school will provide several extended lectures by leading experts on aspects of topological matter. The lecturers will give graduate level presentations introducing to state-of-the-art methods and techniques featuring the key issues of the field of topological matter. The school lectures are complemented by practical sessions on computer implementation of topological properties in numerical codes. While the school is primarily aimed at instructing PhD students and young postdoctoral researchers, more senior scientists who want to acquaint themselves with the subject of the school are also welcome.

Registrations: http://tms16.sciencesconf.org
Deadline 24.03.2016.

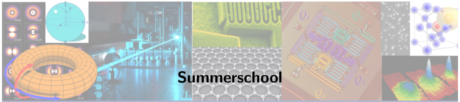
Scientific Coordination:

Dario Bercioux, Donostia International Physics Center (ikerbasque)

M. Reyes Calvo, CIC NanoGune Donostia-San Sebastián (ikerbasque)

Maia G. Vergniory, Donostia International Physics Center

François Konshelle, CFM Donostia-San Sebastián



Nanotechnology meets Quantum Information

July 11 to 14, 2016 — Donostia-San Sebastián

Ever smaller and better designed semiconductor structures are reaching the quantum realm, leading to new promises and challenges in information processing. In the school "Nanotechnology meets Quantum Information" seven leading experts will provide a comprehensive and broad overview about different implementations for both quantum information processing and quantum simulation enabled by recent progress in nanotechnologies and the experimental and theoretical challenges in exploring the prospects of quantum computing, quantum simulation, and the physics of quantum many-body systems.

Information and Registration: `http://nanoqi.dipc.org/`

Application Deadline: April 30th, 2016

School fee: 200EUR

Lectures by:

Darrick E. Chang — Liang Fu — Ataç Imamoğlu — Daniel Loss
J. Ignacio Cirac — Mikhail D. Lukin — Andreas Wallraff

Organization: Géza Giedke (DIPC)
Alejandro González-Tudela (MPQ)
J. Ignacio Cirac (MPQ)
email: nanoqi@dipc.org

Universidad del País Vasco — Euskal Herriko Unibertsitatea

GLOCC transformations of $\gamma(\lambda)$

- step 1: from $(\lambda, \lambda, \lambda)$ we can prepare *any* $(\lambda_1', \lambda_2', \lambda_2')$ with $\lambda \geq \lambda_2' \geq \lambda_1'$ by GLOCC on first mode alone
- step 2: from $(\lambda_1', \lambda_2', \lambda_2')$ by acting on 2nd mode: get only $\det D_{12} \leq 0$ states
- step 3: from $(\lambda_1'', \lambda_2'', \lambda_3'')$ with $\det D_{12} \leq 0$ states, a state with $\det D_{12} > 0$ cannot be prepared
- in contrast, from the shared two-mode squeezed states with $\det D_{23} > 0$ we can obtain all symmetric states (given enough initial squeezing), i.e. we can make the third determinant negative